



**OptiBPM**  
Waveguide Optics Design Software

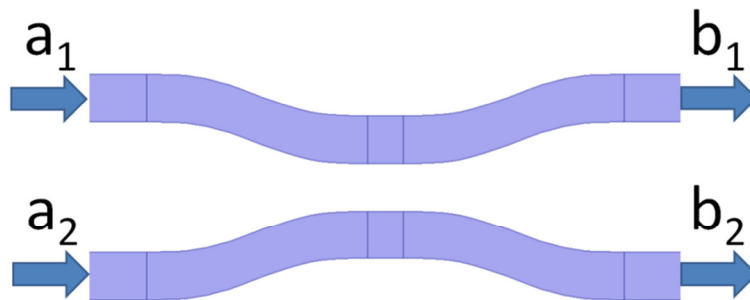
# Symmetric Lossless X Coupler

## Applications

- Basic component in optical circuits
- Mach-Zehnder interferometer
- Waveguide Optical Filters
- PON

## Overview

The X Coupler is a basic component used in many kinds of optical circuits. Here its properties are analysed by theoretical means, and also by detailed simulation of the optical propagation by OptiBPM.



*Fig. 1 The X Coupler. Input optical waves are shown on the left, output waves on the right. The input is represented by complex amplitudes of the fundamental modes of the two waveguides at the inputs,  $a_1$  and  $a_2$ . The output is represented by complex amplitudes  $b_1$  and  $b_2$  of the fundamental mode of the waveguides at the outputs.*

## Theory

Cross coupling can occur when two waveguides are brought close together and interact via the evanescent fields outside their boundaries. This cross coupler is represented by a transfer matrix that has a single parameter,  $c$ .

Maxwell's equations are linear, and therefore the solution of optical field for one set of excitations can be added to the solution for another set to get the total field when both excitations are applied. In terms of the amplitudes  $\mathbf{a}$  and  $\mathbf{b}$ , it means that the output amplitudes will be a linear combination of the input amplitudes:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \mathbf{T} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (1)$$

where  $\mathbf{T}$  is the transfer matrix:

$$\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad (2)$$

In the general case, the complex numbers  $T_{11}$ ,  $T_{12}$ ,  $T_{21}$ , and  $T_{22}$  are all different and independent. However, the X Coupler component considered here is symmetric and loss-less, and this results in a reduction of independent parameters to one real variable whose value is between 0 and 1.

## Consequence of Symmetry

Light that goes from  $a_1$  to  $b_1$  could be defined as the value of  $b_1$  when  $a_1 = 1$  and  $a_2 = 0$ . In this case,  $b_1 = T_{11}$ . If we then set  $a_1 = 0$  and  $a_2 = 1$  the symmetry of Fig. 1 will indicate that the same result will be obtained on  $b_2$  as was obtained in the first case on  $b_1$ . Therefore  $T_{11} = T_{22}$ . By the same argument, we can show that  $T_{12} = T_{21}$ . Therefore the symmetry reduces the four independent coefficients to two. Call the coefficient for crossing from one waveguide to the other X, and the one that stays in the same waveguide Y, the transfer matrix becomes

$$\mathbf{T} = \begin{bmatrix} Y & X \\ X & Y \end{bmatrix} \quad (3)$$

## Consequence of Conservation of Energy

The X Coupler is loss-less. It is supposed that all the optical power that enters on the ports on the left will make its way to the two ports on the right. The optical power at any of the ports is proportional to the square of the magnitude of the optical amplitude. Therefore we must have

$$b_1 \bar{b}_1 + b_2 \bar{b}_2 = a_1 \bar{a}_1 + a_2 \bar{a}_2 \quad (4)$$

where the over-bar denotes complex conjugate. Substituting  $\mathbf{b} = \mathbf{T} \mathbf{a}$  into (4) and collecting like terms of multiples of  $\mathbf{a}$  yields

$$(Y\bar{Y} + X\bar{X} - 1) \bar{a}_1 a_1 + (X\bar{Y} + \bar{X}Y) \bar{a}_1 a_2 + (\bar{X}Y + X\bar{Y}) a_1 \bar{a}_2 + (X\bar{X} + Y\bar{Y} - 1) \bar{a}_2 a_2 = 0 \quad (5)$$

The above equation must be true for any value of  $a_1$  and  $a_2$ . Letting  $a_1 = 1$  and  $a_2 = 0$ , we will find that

$$X\bar{X} + Y\bar{Y} = 1 \quad (6)$$

Having established (6), note that (5) reduces to

$$(X\bar{Y} + \bar{X}Y) \bar{a}_1 a_2 + (\bar{X}Y + X\bar{Y}) a_1 \bar{a}_2 = 0 \quad (7)$$

Now let  $a_1 = 1$  and  $a_2 = 1$ . From this we can conclude

$$X\bar{Y} + \bar{X}Y = 0 \quad (8)$$

## Delay Normalization

The time it takes light to cross the coupler results in a delay. The delay is proportional to the physical length of the coupler. In the model of an optical circuit, the X Component is often normalized so as to have this delay eliminated. To model a real coupler it will be necessary to have a delay line in series with the X Coupler. The delay line will account for the fact that the plane on which the “**a**”s are defined is in a different place than the plane on which the “**b**”s are defined. In this manner of use, the X Coupler itself is modelling only the cross-coupling. Therefore in the definition of the X Coupler, we are at liberty to define the coefficient, Y, to be a real number. (Y is also almost independent of  $\lambda$ .) Y is the fraction of light that stays in the same waveguide after going through the coupler. We normalize it to be a positive real number, and from conservation of energy, Y must therefore be between 0 and 1. Y can be represented instead by another positive real number, c, defined as

$$Y = \sqrt{1 - c} \quad (9)$$

## X Coupler Transfer Matrix

With the above definition, (6) implies

$$|X|^2 = c \quad (10)$$

Since Y is taken to be a real number, (8) implies that the real part of X is 0. Therefore there are two possibilities for X:  $X = j\sqrt{c}$  or  $X = -j\sqrt{c}$ .

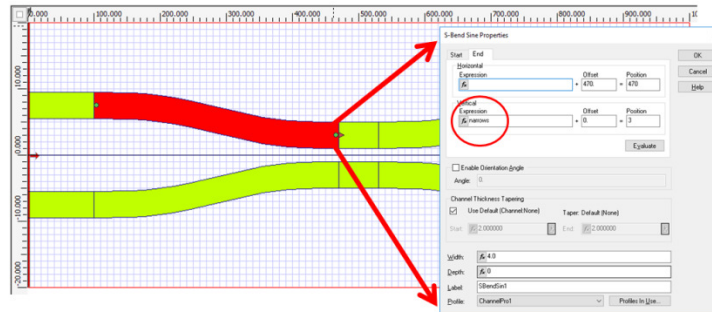
Therefore the transfer matrix for the X Coupler is

$$\mathbf{T} = \begin{bmatrix} \sqrt{1 - c} & jp\sqrt{c} \\ jp\sqrt{c} & \sqrt{1 - c} \end{bmatrix} \quad (11)$$

Where c is the coupling coefficient. It is a real number between 0 and 1. p can take the value +1 or -1, it depends on the details of the coupler.

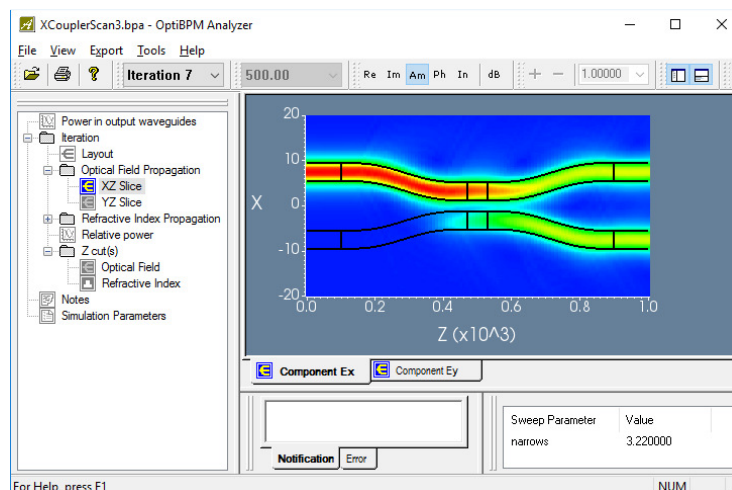
## Simulation by OptiBPM

OptiBPM can be used to design optical waveguides for the X Coupler. The horizontal axis is the propagation axis, Z. The vertical red line (corresponding to Z = 0) marks the beginning of the simulation by the Beam Propagation Method. The BPM is used to calculate the propagation resulting from a modal excitation found on the Input Plane, the vertical red line on the left. Propagation proceeds from left to right. Numerical experiments show that for the given waveguide design, the propagation of the waveguides is independent when separated by the initial distance, 15  $\mu\text{m}$ .



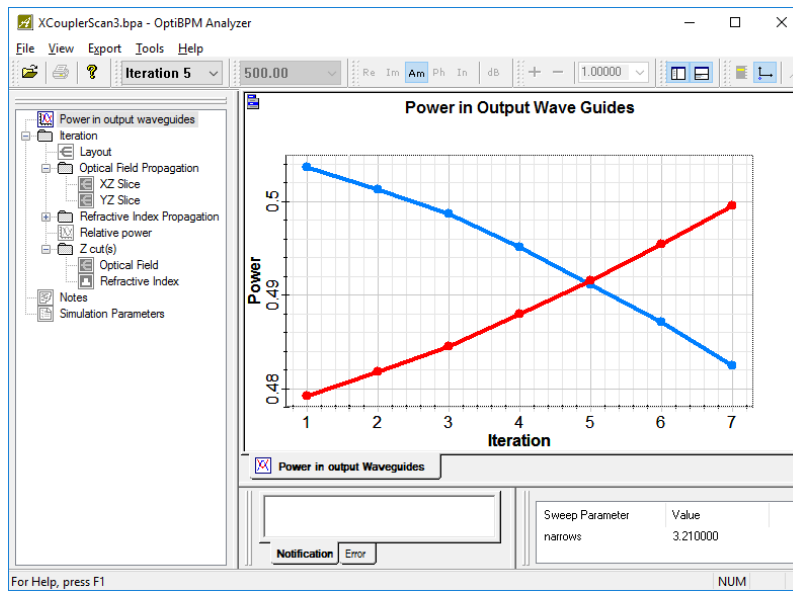
**Figure 2, Layout of the Symmetric X Coupler. Positions of some waveguides are set by the user parameter “narrows”**

If the waveguides are brought closer together, the light can cross from one waveguide to the other. The fraction that crosses depends on the proximity and the distance the waveguides remain close. Any desired fraction of coupling can be achieved by adjusting the distance between them. In OptiBPM a parameter can be created, here called “narrows”. This parameter can be used in calculating the coordinates of the six waveguides affected when this parameter is changed. This parameter can also be scanned, in which the simulation is repeated for a range of the parameter. The requested simulation results are recorded and plotted for each simulation. Figure 3 shows results for iteration 7, in which “narrows” was 3.22  $\mu\text{m}$ .

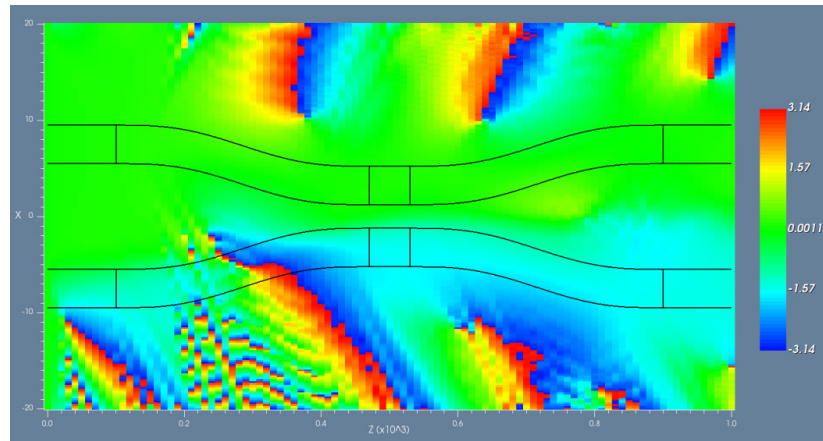


**Figure 3, Results for iteration 7 of the simulation. At this iteration, the parameter “narrows” was 3.22  $\mu\text{m}$**

OptiBPM will also plot the power in the two output waveguides for each iteration, as shown in Figure 4.



**Figure 4, Modal Power in the two output waveguides for different values of the parameter. The 3dB point is found at iteration 5 when “narrows” is 3.21  $\mu\text{m}$ .**



**Figure 5, The optical phase at iteration 5.**

Instead of plotting the amplitude of the optical wave, as in Figure 3, the optical phase can be plotted, as in Figure 5. The excitation for Figure 5 is the same as in Figure 3,

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

From (11), the following output is expected

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \sqrt{1-c} \\ jp\sqrt{c} \end{bmatrix}$$

The OptiBPM simulation predicts the same power at  $b_1$  and  $b_2$ , both slightly less than -3 dB. The phase of  $b_2$  is observed to be  $-\pi/2$ . This can be interpreted as the result from (11) with  $c = 1/2$ , in which a small (0.1 dB) loss is applied as well as the cross coupling. It appears that  $p = -1$ , although there is nothing in the theory (considerations of symmetry or conservation of energy) to indicate that the value should be -1, and not 1. The correct choice of  $p$  seems to be available from the detailed numerical simulation only.